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The Institute of Automotive Engineer Assessors

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Basic Principles of Maths and Physics Applicable to Accident Reconstruction

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BASIC PRINCIPLES OF MATHS AND PHYSICS APPLICABLE TO ACCIDENT RECONSTRUCTION

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SEQUENTIAL INDEX

1. SYMBOLS, UNITS AND CONVERSION FACTORS

S. I. Units

Symbol	Quantity	Unit
$\bar{v}$	average velocity	metres per second (ms^{-1})
u	initial velocity	metres per second (ms^{-1})
v	final velocity	metres per second (ms^{-1})
s	distance	metres (m)
t	time	seconds (s)
a	acceleration	metres per second per second (ms^{-2})
g	acceleration due to gravity	$9.81ms^{-2}$
μ	coefficient of friction	(none)
m	mass	kilograms (kg)
W	weight	newtons (N) or kilograms force (kgf)
F	force	newtons (N) or kg ms^{-2}
ρ	momentum	kilograms metres per second (kg ms^{-1})
R	range (trajectories)	metres

Circular movement

r	radius	
M	mid–ordinate	
C	chord	

Other Units

mph	speed	miles per hour (mph)
kmh^{-1}	speed	kilometres per hour (kmh^{-1})

Conversion factors

Velocity

mph to ms^{-1}	x 0.447
ms^{-1} to mph	÷ 0.447
mph to kmh^{-1}	x 1.61
kmh^{-1} to mph	÷ 1.61
ms^{-1} to kmh^{-1}	x 3.6
kmh^{-1} to ms^{-1}	÷ 3.6

Length

miles to kilometres	x 1.61
kilometres to miles	÷ 1.61
metres to feet	÷ 0.3048
feet to metres	x 0.3048
inches to millimetres	x 25.4
millimetres to inches	÷ 25.4

General information

Speed of sound	330 ms^{-1}

GREEK LETTERS AND NON-STANDARD CHARACTERS

Symbol name	Greek alphabet Lower case	Upper case	Roman alphabet
alpha	α	A	a
beta	β	B	b
delta	δ	Δ	d
phi	ϕ	Φ	f
mu	μ	M	m
pi	π	Π	p
theta	θ	Θ	q
rho	ρ	P	r
omega	ω	Ω	w
psi	ψ	Ψ	y

When using '*Microsoft Word*', Greek letters are entered by changing to *Symbol* font and entering the equivalent Roman letter. By convention they are normally italicised.

OTHER MATHEMATICAL SYMBOLS

$<$	Less than
$>$	Greater than
$\approx$	Is more or less equal to
$\propto$	Is proportional to

NON-STANDARD CHARACTERS

When producing accident investigation reports it is often necessary to use characters that are not available on a standard keyboard. There are many ways of entering these characters but one quick way is shown below. It is applicable to any 'MS Windows' program.

It is recommended that, when using Microsoft Word, complex equations be entered using the Equation Editor applet which is accessible using Insert – Object – Microsoft Equation Editor. It is strongly recommended that the standard shortcut button to this applet be placed on one of the toolbars.

To produce the character in the table below hold the ALT key down and type the numbers to the right of the symbol using the numeric keypad with NUM LOCK enabled. Many more characters are available using this feature but only those applicable to accident investigation are shown in the table.

°	0176	
±	0177	
^{2}	0178	
µ	0181	
$\propto$	0181	(Symbol font)
$\approx$	0187	
¼	0188	
½	0189	
¾	0190	
ϕ	0198	
÷	0247	

To produce superscript characters press CTRL + SHIFT + PLUS SIGN followed by the character(s) required.
To produce subscript characters press CTRL + EQUAL SIGN followed by the character(s) required.
(Ideally the standard shortcut buttons for these two features should be placed on one of the toolbars)

DEFINITIONS

Acceleration
The rate of change of velocity with respect to time.
It is a vector quantity.
Units: metres per second per second (ms^{-2})

Acute angle
An angle of less than 90°

Centre of Gravity
The fixed point through which the resultant force of gravity always passes irrespective of the position of that body

Coefficient of friction
The ratio of the frictional force to the force pressing the two surfaces together. Units: None

Conservation of linear momentum
When two or more bodies act upon one another, their total momentum remains constant provided no external forces are acting upon them

Cosine of an Angle
In a right–angled triangle, the cosine of a given angle is the numerical ratio between the side adjacent to the angle and the hypotenuse. $\text{Cos} = \frac{\text{adj}}{\text{hyp}}$

Displacement
The linear distance between two defined points in a specified direction. It is a vector quantity. Unit: metre (m)

Distance
The total length in a specified path, straight or otherwise. It is a scalar quantity. Unit: metre (m)

Dynamic Friction
The friction that exists between two surfaces in contact after movement between the two has started.

Force
That which tends to change a body's state of rest or its uniform motion in a straight line. Unit: newton (N)

Friction
The force which opposes the relative sliding motion of two surfaces in contact with one another.

Gravitational Force
The force of attraction between all objects that tends to pull them toward one another. (See Newton's Law of Universal Gravitation below).

Inertia
The property of matter that causes it to resist any change of motion in either direction or speed.

Laws of Friction
The frictional force is proportional to the force pressing the two surfaces together.
The frictional force is independent of the contact area.
The frictional force depends upon the nature of the two surfaces in contact.

Limiting Friction
The maximum value of static frictional force immediately before one surface moves relative to another with which it is in contact.

Mass
The amount of matter contained in a body. It is a scalar quantity and remains constant. Unit: kilogram (kg)

Momentum
The product of the mass of a body and its velocity. It is a vector quantity. Unit: kilogram metre per second (kg ms–1)

Newton's 1st law of motion (Law of inertia)
A body will continue in a state of rest or of uniform motion in a straight line unless compelled by some external force to do otherwise.

Newton's 2nd law of motion
The rate of change of momentum of a body is proportional to the force applied and takes place in the direction in which the force acts. Therefore $\frac{F}{W} = \frac{a}{g}$

Newton's 3rd law of motion — To every action there is an equal and opposite reaction.

Newton's Law of Universal Gravitation — Any two particles of matter attract one another with a force which is proportional to the product of their masses and inversely proportional to the square of their distance apart.

newton — The force required to accelerate a mass of 1kg at a rate of $1ms^{-2}$. Symbol (N). (Note: 9.81N = 1kgf. kgf is not an S I unit)

Obtuse Angle — An angle exceeding 90° but not greater than 180°.

Reflex Angle — An angle greater than 180°.

Scalar Quantity — That which has size or magnitude only.

Sine of an Angle — In a right–angled triangle, the sine of a given angle is the numerical ratio between the side opposite the angle and the hypotenuse. $\text{Sin} = \frac{\text{opp}}{\text{hyp}}$

Speed — The rate of change of distance with respect to time. It is a scalar quantity. Units: metres per second (ms^{-1}) or miles per hour (mph) or kilometres per hour (kmh^{-1})

Static Friction — The friction existing between two surfaces in contact and which are stationary relative to each other.
The limiting static frictional force is almost always higher than the frictional force produced once an object has started to move.

Tangent of an Angle — In a right-angled triangle, the tangent of a given angle is the numerical ratio between the side opposite the angle and the adjacent side. $\tan = \frac{\text{opp}}{\text{adj}}$

Uniform Acceleration — A body is said to be uniformly accelerating when its rate of change of velocity with time is constant. It is a vector quantity. Unit: metres per second per second (ms^{-2})

Uniform Velocity — A body is said to have uniform velocity when its rate of change of displacement remains constant. It is a vector quantity. Unit: metres per second (ms^{-1}) or miles per hour (mph), kilometres per hour (kph^{-1})

Vector Quantity — That which has both magnitude and direction.

Velocity — The rate of change of displacement with respect to time. It is a vector quantity. Unit: metres per second (ms^{-1}). miles per hour (mph). kilometres per hour (kmh^{-1})

Weight — Weight is the force (due to gravity) which a body exerts upon anything which freely supports it. It is a vector quantity. Unit: - newton (N) (Note: (9.81N = 1kgf. kgf is not an S I unit)

ARITHMETIC

The signs '+', '–', 'x' and '÷' are the basic arithmetic symbols. The correct term for these signs is ***operators***. The number, or letter, on which the operation is being performed, is called the ***operand***.

The sign '+' relates to a positive number, *eg*, '+ 5' *(Note. When dealing with positive numbers it is usual to omit the '+' sign)*. The sign also relates to the operation of **adding** a sequence of numbers. This operation is also referred to as '**addition**', '**sum**', '**plus**', *eg*, '3 + 2' means '3 plus 2' or 'the sum of 3 and 2'.

The sign '-' relates to a **negative** number, *eg*, '-5'. The sign also relates to the operation of **subtracting** one number from other numbers. This operation is also referred to as '**deduct**' or '**minus**' or '**take away from**', *eg*, '3 - 2' means '3 minus 2' or '3 take away 2'.

The '-' sign is known as a minus sign so that '-5' is said to represent 'minus 5'.

The sign 'x' relates to the operation of the multiplication of a series of numbers, *eg*, '3 x 2' means '3 multiplied by 2'. This is also referred to as 'times'. The result of this operation is known as the 'product'. Multiplication is a quick or shorthand form of addition *eg*, '3 x 3' means the same as '3 + 3 + 3' (ie, 3 lots of 3). The operation of multiplication can also be indicated by the use of parentheses (brackets) *eg*, '(3)(2)' means the same as '3 x 2'. Another method of expressing a special kind of multiplication is by the use of **powers** *eg*, '3^2' (where the figure '^{2}' is called a 'power') means the same as '3 x 3'.

The sign '÷' relates to the operation of **division** of a series of numbers. The expression '*division*' means '*How many times can one number be taken away from another number?' eg*, '9 ÷ 3 = 3' means that '3' can be taken from '9' '3' times. Hence, just as multiplication is a short form of addition, division is a short form of subtraction. The operation of division can also be expressed in other ways.
eg, 9 ÷ 3 can be written as $\frac{9}{3}$ or 9/3.

Rules for addition and subtraction

The rule for addition is:

i. **With like signs**, the sum of the absolute values with the common sign prefixed.

+4	-5
+	+
+3	-4
+7	-9

Another way to look at the problem is that you have £4 (ie, +£4) and a friend gives you another £3 (ie, +£3). You now have a total of £7 (ie, [+£4] + [+£3] = £7).
In the second example you owe one friend £5 (ie, –£5) and another friend £4 (ie, –£4).
You owe a total of £9 (i.e [–£5] + [–£4] = –£9)

ii. **With unlike signs** the result becomes the difference between the absolute values prefixed by the sign of the greater absolute value.

-3	-9	-9	+2
+	+	+	+
+5	-4	+3	-7
+2	+5	-6	-5

In the first example, you owe a friend £3 but he gives you £5. You now have +£2. In the second example, you have £9 and you give a friend £4. You now have +£5.

Consider the other examples above in a similar fashion.

The rule for subtraction is:

Change the sign of the subtrahend and proceed as for addition.
*Note: The **subtrahend** is the figure to be taken away.*

+7	–6	–3	–8	+7
–	–	–	–	–
+3	–2	–7	+3	–2
+4	– 4	+4	–11	+9

In the first example, you have £7 and you take away £3. You now have £4.

In the second example, you think you owe £6 but there was a mistake and you owe £2 less. You now owe £4.

Consider the other examples above in a similar fashion.

Multiplication and division of numbers

The rules for multiplication and division of positive and negative numbers are as follows:

(+) x (+) = (+)	+ 2 x + 3 = + 6
(+) x (–) = (–)	+ 2 x – 3 = – 6
(–) x (+) = (–)	– 2 x + 3 = – 6
(–) x (–) = (+)	– 2 x – 3 = + 6

BODMAS (Bodmas)

Sometimes the order in which an operation is performed on a series of numbers is important.

Consider the following: 2 + 5 + 1 + 8

These numbers can be added in any order and the answer will always be 16.

But, what about subtraction? Can the same rules apply?

eg, 8 – 4 + 2

Is the answer 6 or is it 2? It depends which part of the calculation is performed first.

Without specific rules to follow there would be no 'correct' answer because the problem is ambiguous. It should be clarified by the use of brackets.

eg, a) 8 – (4 + 2) = 2 or b) (8 – 4) + 2 = 6

Note: When brackets are used, the terms inside should be evaluated before involving the terms outside the brackets.

From the foregoing, a rule can be developed as follows:

Subtraction of a sequence of numbers grouped inside a bracket changes the signs inside the brackets *ie*, all '+' signs inside the bracket change to '–' and all '–' signs change to '+'.

eg, 8 – (4 + 2) = 2 becomes 8 – 4 – 2 = 2

Note: The rule does not apply to the second example above ie, '(8 – 4) + 2 = 6' because the numbers grouped inside the brackets are <u>to be added</u> to the number outside the bracket ie, the equation could also be written as + (8 – 4) + 2 = 6.

Using the same rule, if it is desired to enclose certain terms inside a bracket and the bracket is preceded by a '–' sign, the signs that fall inside the bracket must be changed.

eg, 10 – 2 – 6 = 10 – (2 + 6) = 2

'Bodmas' is an acronym and a mnemonic which gives the order for working out complex calculations. Most modern calculators comply with the rules of 'Bodmas'.

Brackets
Order
Division
Multiplication
Addition
Subtraction

Rules:

1. Work out any part inside brackets first; then calculate any squares or square roots or other powers ('*Orders*').

2. Work out any division or multiplication expressions.

3. Work out addition or subtraction expressions.

4. If the whole expression is above and below a horizontal line, treat the problem as a division, with brackets around the top and bottom halves.

Example: 2 + 3 x 4 =
If an attempt to solve this manually is made by dealing with each operator in sequence the answer 20 will be obtained. (2 plus 3 equals 5, times 4, equals 20). This is incorrect. Using the rules of 'Bodmas', multiplication is dealt with first ie, 3 times 4 and then add the 2: the correct answer is 14. A calculator would give the correct answer. Further examples of dealing with 'Bodmas' are shown under the section dealing with the use of calculators.

Decimal numbers

In arithmetic, counting is performed from RIGHT TO LEFT in columns using units, tens, hundreds, thousands etc, *eg*,

Thousands	Hundreds	Tens	Units

The basic rule is that a count of 10 in the units in one column is the same as a count of 1 in the tens column (or the next column to the left), *eg*,10 x 1 = 1 x 10

Sometimes it is necessary to express numbers smaller than 1 and, in the decimal system, this is done by creating extra columns to the right of the units column

eg,

Thousands	Hundreds	Tens	Units	Tenths	Hundreths	Thousandths

The decimal point (.) shows that a column to the right of the units column is being considered. So, for example, 2.1 means '*two units and one tenth*' which is expressed as "*two point one*".

0.2 means '*no units and two tenths*' which is stated as "*nought point two*".

If there is no number after the decimal point, neither the decimal point nor a zero after the number are shown unless it is required to show the degree of precision of the number,

eg, the number '*Two*' is written as '2'. However if it expressed to 1 decimal place it is written as 2.0.

If there are no numbers to the left of the decimal point, a zero and the decimal point are always shown,

eg, the number '*nought point two*' is written as '0.2'.

Powers, squares and square roots

'*Powers*' are a short way of writing a chain of multiplications, *eg*, '3 x 3' can be written as '3^2', meaning two '3s' multiplied together.

In the example, the small superscript number '2' tells us to '*square*' the '3'. The small number is called a **power**. Alternative names are an **index** or an **exponent**. (The number itself is called the **mantissa**). In other words, '3' squared is the same as '3' raised to the power of 2.

In general, 4^3 ('4' raised to the power of '3') means '4' multiplied by '4' multiplied by '4' (or three '4s' multiplied together),
eg, $5^4 = 5 \times 5 \times 5 \times 5 = 625$
Note: any number raised to the power of '^{0}' is 1.

When numbers (involving the same 'base') and raised to different powers are multiplied their indices are added.

When numbers (involving the same 'base') and raised to different powers are divided their indices are subtracted,

eg, $2^2 \times 2^3 = 2^5$ and $2^{-2} \times 2^4 = 2^2$ and $3^6 \div 3^4 = 3^2$

A number raised by a negative power *eg*, 3^{-2} is the same as $\frac{1}{3^2}$. This will be frequently used when dealing with units of measurement used in accident investigation.

A number raised by a fraction *eg*, '$9^{1/2}$' is the root of that number. The number '2' in the '½' indicates that in this case the square root is required. The square root of 9 is the number which, when multiplied by itself, generates 9.

If the fraction were '⅓' it would indicate that the cube root of the number is required, *eg*, '$64^{1/3}$' is the number which, when multiplied by itself three times generates 64, ie, 4 x 4 x 4.

A more common way to indicate the root of a number is to use the radical sign ie, $\sqrt{4}$. This is the preferred and recommended method for the purposes of accident investigation. It is important that the horizontal extension line covers all the numbers subject to the square root.

eg, $\sqrt{256}$ would be 16 whereas $\sqrt{25}\ 6$ would technically be 30 (the square root of 25 times 6, ie, 5 x 6).

Note: When using a radical sign it is convention that, if no number appears before the sign the sign indicates the square root of the number. In all other cases the number must be included with the root sign eg, $\sqrt[3]{8}$ *means the cubed root of 8, ie, 2.*

A third way is to use the symbol followed by the number in parentheses
eg, √(256) would be 16 and √(25)6 would be 30.

In the section, dealing with the multiplication and division of numbers it has been shown that a negative number multiplied by a negative number gave a positive answer, *eg*, –7 x –7 = 49.

It follows that the square root of a number could be a positive or a negative number, *eg*, $\sqrt{49}$ = 7 or –7.

However, it is not possible to find the square root of a negative number.

Try $\sqrt{-49}$ on a calculator, an error message will be displayed.

Scientific Notation

Scientific notation is a type of shorthand. It is most often used to express very large or very small numbers. Very large or small numbers such as these are not frequently met within accident investigation but it is necessary to be able to recognise a number presented in scientific notation and to understand its meaning.

An example of such a number is 3×10^7. If written in its full form the number would read 30,000,000. The number ie, the '3' is known as the **mantissa**. It is always a number between 1 and 10. It is always multiplied by 10 (the **base**) which itself is raised to some power (the **exponent**). The exponent shows how many decimal places zeros move left or right to make that number, if the number were to be written out in full.

If the full number was 38,600,000 then in scientific notation it would be written as 3.86×10^7. This indicates that there are seven numbers after the '3', and that the first two are '86'; the remainder must be zeros.

The principal is the same for very small numbers, for instance 3.86×10^{-7} would be 0.000000386 *ie*, place the decimal point seven places to the left of the original decimal point position. (Remember to always put a further zero in front of the decimal point when dealing with a value less than 1).

Decimal Places

Sometimes when performing calculations an answer with many decimal places can be achieved. For example the value of π when expressed to 11 decimal places is 3.14159265359.

To express this number to 2 decimal places merely count that number of places to the right of the decimal point and then round off that last digit. This process is also known as rounding.

Figures 5 and above cause the immediately previous figure to be rounded up.

Figures *less than* 5 cause the immediately previous figure to be rounded down.

Thus π becomes 3.14. To 3 decimal places it would be 3.142.

Significant Figures

When calculations are performed, answers containing many numbers may result. But how many of those numbers are relevant?

For example, 8 sweets are to be shared between 7 people. How many sweets does each person get? The answer is obviously 1. Use of a calculator is unnecessary for this problem. Now suppose £8 was to be distributed between 7 people. Use of a calculator will give an answer of 1.1428571. The best that could be done would be to give each person £1.14 and keep the 2p change. In other words, only the first three numbers are significant; the last five numbers are too small to be significant. However suppose you were one of seven people who had won the lottery, where the prize was £8,000,000. Under these circumstances it would probably be deemed that more numbers would be significant otherwise the 'banker' would gain an additional £20,000. In this example, the last 1 in the number (1.1428571) represents 1p. If you had just won such a large amount of money you would probably agree that 1p is insignificant. In this case, you might only consider the first 7 significant figures ie, 1.142857.

The speed of sound, at 0° C, is 331.6 metres per second. It would be nonsense in practical scenarios to perform a calculation and use this figure because the speed of sound varies with changes in temperature. This number might therefore be expressed to 2 significant figures in which case it would be 330 metres per second. Expressed to 1 significant figure it would be 300 metres per second.

Similarly, if a speed were calculated to be 43.367mph the decimal fraction would be insignificant and the answer should be expressed to 2 significant figures *eg*,, 43mph.

Note:
1. The first significant figure is the first digit that is not a zero. In other words, the first significant figure in 0.000004356 is the 4.
2. Once the first significant figure is found, all the remaining digits are significant, whether they are zero or not.

Fractions

'*Fraction*' means '*part of*'. A fraction of something is a part of the whole. It is the result of separating the whole object or number into equal parts. Fractions may be written as either '***vulgar fractions***' or '***decimal fractions***'. Initially let us deal with vulgar fractions, *eg,* ½ is the whole thing (or the number '1', which represents '*a whole thing*') divided into two equal parts of which only 1 is being considered. In the case of ¾ the number is divided into four equal parts of which three are being considered. The numbers above the line of a fraction are called the **numerator** and those below the line the **denominator**.

Adding Fractions

It is only possible to directly add and subtract fractions that have a common denominator, *eg,* It is not possible to directly add ½ to ⅔.

It is necessary to find a '*common denominator*' and, to keep the working as simple as possible, for the smallest number that will serve this purpose. This number is known as the **lowest common denominator**. This, when only two vulgar fractions are involved, is found by multiplying each fraction by the denominator of the other,

$$\frac{1}{2}+\frac{2}{3} = \frac{3}{6}+\frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Multiplying Fractions

To multiply fractions simply multiply their respective numerators and denominators,

$$\frac{1}{2}\times\frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

Dividing Fractions

To divide fractions take the **reciprocal** (the inverse) of the second fraction and continue as for multiplication,

eg, $$\frac{2}{3}\div\frac{1}{2} = \frac{2}{3}\times\frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$$

When a fraction has a numerator larger than the denominator such as ⁸⁄₅ it is known as an improper fraction. These are normally expressed as a whole number followed by a proper fraction as in the example above.

Decimal Fractions

To convert a vulgar fraction into a decimal form, just divide the numerator by the denominator,

eg, $$\frac{3}{4} = 0.75$$

A decimal fraction is a fraction with a denominator of powers of 10. Just think about the number 0.5. What it actually means is 'five tenths' $\left(\frac{5}{10}\right)$, which is the vulgar form for the same value. So to convert a decimal fraction into a vulgar fraction simply divide by the appropriate denominator and simplify,

eg, $0.75 = \frac{75}{100} = \frac{3}{4}$ and $0.625 = \frac{625}{1000} = \frac{5}{8}$

Ratios and proportion

A ratio is a comparison between two similar quantities. The word **proportion** can often be interchanged with **ratio**. It is another way of expressing a fraction. Two people have £100 between them. One has £60. The other, therefore, has £40. It can be said that the £100 is shared in the proportion or ratio of 6 : 4 which could be simplified to 3 : 2 or 1.5 : 1. The first person has 60% of the £100, the second has 40% of the £100.

In a car gearbox the ratio of the first gear might be 5.7:1. This would mean that for every 5.7 turns of the engine crankshaft (input turns), the output shaft from the gearbox will turn once. Second gear might have a ratio of 2.9:1, so for every 2.9 turns of the engine the output shaft will turn once. The effect is that, for a given number of crankshaft revolutions, the car will move at a faster speed. In a case such as this it would be nonsense to talk about a total, it is merely the relationship between the two numbers that is being considered.

An example of where ratios are used is in the production of scale plans. If it were possible to draw a plan full size then the ratio would be 1:1. To make the plan an appropriate size for the paper it may be appropriate to draw it at one hundredth of its true size. In this case one centimetre on the plan would represent 100cm (1 metre) on the ground and this would be a ratio of 1:100. If it were needed for the plan to fit onto a smaller sheet of paper it might be drawn at half that size in which case the scale would be 1:200 ie, 1 centimetre is equal to 2 metres.

Direct proportion is when there is a simple multiplying connection between two things – as one increases so does the other. An example would be the relationship between distance and time. Imagine a car travelling at an average speed of 30mph. If it took 30 minutes to cover a particular distance, by using the equation $s = \bar{v} \times t$ one would be able to calculate the distance (15 miles). If it had taken 1 hour then the distance would be 30 miles. Therefore it can be said that the time is directly proportional to the distance. This is expressed in the manner of $s \propto t$ (where the sign $\propto$ means 'is directly proportional to').

In the above example, the relationship only holds while the speed of the vehicle is constant. Another way of showing this relationship is $s = k \times t$ (where k is the constant of proportionality)

In this case, the value of k is the same as the value of $\bar{v}$. Therefore $\bar{v}$ could be substituted for k which would of course produce the original equation.

Example. If $s \propto t$ and 's' = 15 and '**t**' = 5, what is the constant of proportionality?
Answer. If $s \propto t$ then $s = k\,t$
Substitute the known values $15 = k \times 5$ $\quad k = 3$

Inverse proportion is when there is a dividing connection between two things, so that as one increases, the other decreases. For example on a journey, if there is an increase in speed then there will be a decrease in time taken. Therefore, the time taken for a journey is inversely proportional to the average speed.

This can be written as $\bar{v} \propto \frac{1}{t}$ or $\bar{v} = \frac{k}{t}$
In this example, k is the value of the distance 's'.

Percentages

A percentage is a means of expressing a fraction, proportion or rate but always in the form of how many parts of a whole, and where the whole is represented by a numerical value of 100, *eg,* 50% is 50 hundredths or 50/100 which is the same as a half.

To convert a vulgar fraction to a percentage multiply the vulgar fraction by 100 and complete the division.

eg, $\frac{3}{4} \times \frac{100}{1} = \frac{300}{4} = 75\%$

To convert a decimal fraction to a percentage multiply by 100 (ie, move the decimal point two places to the right),
eg, 0.75 expressed as a percentage is 0.75 x 100 = 75%

To express a percentage as a vulgar fraction use 100 as the denominator,

eg, 75% is $\frac{75}{100} = \frac{3}{4}$

To express a percentage as a decimal fraction divide by 100 (ie, move the decimal point two places to the left).
eg, 2.5% becomes 2.5 ÷ 100 = 0.025

CONCEPTS OF ALGEBRA

In algebra, quantities (some of which are unknown) can be represented by letters, or letters and figures. A single expression, containing both a letter and a figure is called a **term**.

Examples.	$5a$	meaning	$5 \times a$
	$4y$	meaning	$4 \times y$
	u	meaning	$1 \times u$

The numeric part of a term is called the **coefficient**. The letter part of the term is sometimes called the **variable** or the **parameter**. Using our example above, the coefficient of '*a*' is 5, the coefficient of '*y*' is 4 and the coefficient of '*u*' is 1.

Some Rules to Remember

$$a + a = 2a$$
$$a \times b = b \times a = ab = ba$$
$$a \times 0 = 0 \times a = 0$$
$$y \times y = y^2$$
$$a \div 4 = \frac{1}{4}a \text{ or } \frac{a}{4}$$

$a(y + z)$ means that y and z must first be added and then the answer multiplied by '*a*'. To expand the bracket, every term inside the bracket must be multiplied individually by the terms outside the bracket. In other words $a(y + z) = ay + az$

In algebra, there are two types of terms known as like and unlike terms and there are rules governing their treatment.

Like terms have the same parameter. Example – $4ab$ and $\frac{5}{8}ab$ are like terms.

Unlike terms have different parameters. Example – $2ay^2$ and $2a^2y$ are unlike terms.

The coefficients of like terms can be added together to simplify a piece of algebra.

Example $4a + a = 5a = (4 + 1 = 5)$

$5a + 2y - 3a = 2a + 2y = 2(a + y)$

When a common factor is taken out of a number of terms, such as the coefficient 2 in the above, this is called factorising.

Equations, Transposition of Equations

Before considering the transposition of equations, consider what is meant by the expression – '*equation*'

All that is being conveyed is that one thing is equal to another. It is irrelevant in what terms the 'thing' is put. The same can be said whether referring to numbers, letters etc.

However, it would be a nuisance to have to write out '*is equal to*' every time a term is expressed in this way, so a symbol replaces that expression (ie, the equality sign '=').

Take a simple equation, using numbers for the terms 3 + 4 = 2 + 5

That expression simply states that 3 + 4 is equal to 2 + 5. That is true because they both add up to 7. So every time 3 + 4 is used, 2 + 5 could be put in its place.

For example, using the same example as a base 3 + 4 + 6 = 13

Replacing the '3 + 4' with '2 + 5' the equation becomes '2 + 5 + 6 = 13'

Consider $a + b = c + d$

If that were true, then wherever '$a + b$' appears, it could be replaced by '$c + d$'.

Transposition of equations/formulae simply means the rearranging of an equation to isolate a term, other than that already isolated in the original equation.

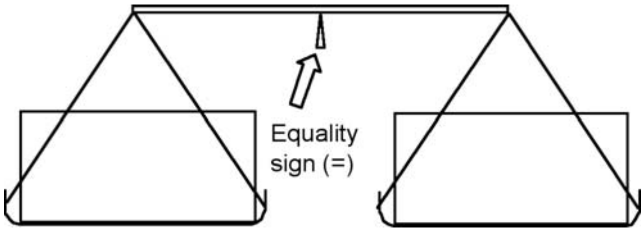
In accident investigation, four basic equations of motion and a number of equations for other purposes are used. The following is a small selection, *eg,*

$v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \left(\frac{u+v}{2}\right)t$ $\quad r = \frac{C^2}{8M} + \frac{M}{2}$

Equations which would cater for every term could easily be derived but this would mean having to be able to recall no less than

16 equations just to cater for the basic terms. It is far easier to learn how to transpose equations and only remember four or five.

By way of an analogy, the equality sign could well be compared with the knife-edged strut of a balance beam, with the two load–pans representing the space either side.



In order to maintain the balance (the equality), whatever is contained in the left–hand pan must possess the same (in this case) weight as that in the right–hand pan. If that does not happen then the balance will not be maintained and the '*equality*' will be false.

If the contents of the left–hand pan are halved then so must be the contents of the right-hand pan. If the right–hand pan is added to, then the left–hand pan must have the same amount added to maintain the balance.

Whatever is done to the one side must also be done to the other. That is the '*Golden Rule*' to retain the balance. That is also the '*Golden Rule*' when changing the position of (transposing) terms and expressions within mathematical equations.

Much of the difficulty experienced in transposition stems from (i) an inability to recognise the object of the exercise and then (ii) failing to implement the rules of '*BODMAS*' and the '*Golden Rule*' above.

For example, from the equation $r = \frac{C^2}{8\,M} + \frac{M}{2}$ it is required to make '*C*' the subject. Application of the '*BODMAS*' rule should alert the operator to the fact that $\frac{C^2}{8\,M}$ is one expression and that $\frac{M}{2}$ is another. In their present form they are quite separate entities and should be dealt with as such.

In the equation [*a* = *b* + *c*], each letter is a separate expression and the '*b*' can be likened to $\frac{C^2}{8\,M}$ whereas '*c*' might compare with $\frac{M}{2}$.

If the original question had been posed as follows 'If 15 = 10 + 5, what does 10 equate to?' the student would instantly recognise that 10 = 15 – 5. That almost sub–conscious action included recognising the object to be achieved, the application of the '*BODMAS*' principle (in simple form) and the application of the '*Golden Rule*' of transposition.

The question was fairly straightforward. The object was to arrive at a position where '10' appeared on one side of the equality symbol and everything else was on the other side of the same sign.

Perhaps it felt more comfortable to express the answer in the form 10 = 15 – 5 ie, $b = a - c$ rather than 15 – 5 = 10 ie, $a - c = b$ but the two expressions are identical in their outcome.

In transposing the equation to isolate the '10', the '5' (which was originally being added to the right–hand side) must be subtracted from that side. That would certainly leave the '10' on its own but, if taken no further, would leave the expression [10 = 15], which is patently incorrect.

That is because '*Golden Rule*' has not been implemented. Whatever is done to one side MUST be done to the other. As the '5' was subtracted from the right–hand side, it must also be subtracted from the left–hand side. In which case the changed (transposed) equation becomes [10 = 15 – 5]. That is patently correct.

Whilst all of the foregoing is simple, straightforward and probably blindingly obvious, if the act of transposition is carried through one step at a time, slowly and carefully, the operation gets no more difficult.

WARNING:

Failure to implement the '*Golden Rule*' can seriously damage the chance of obtaining the correct answer!

The rules, methods, logic and algebraic processes remain the same no matter how complicated the transposition may initially appear.

The following are some slightly different examples.

Consider the equation [$a \times b = c + d$], and where the task is to make 'b' the subject. This time, as it is ($a \times b$) it is necessary to divide the left–hand side by 'a' to isolate 'b'.

But the 'Golden Rule' states DO THE SAME TO BOTH SIDES and so: $\frac{a \times b}{a} = \frac{c+d}{a}$ which leaves $b = \frac{c+d}{a}$

If an equation appears difficult to rearrange, write each step down separately.

In order to carry out calculations, letters in an equation are simply replaced by (substituted by) numbers.

Taking a simple equation composed of letters [$a + b = c + d$], if it is known: 'a' = 3, 'c' = 4 and 'd' = 5 and it is required to find the value of 'b', first isolate 'b' by subtracting 'a' from both sides $a + b - a = c + d - a$ leaving $b = c + d - a$ then substitute the numbers for the letters $b = 4 + 5 - 3 = 6$

Check the answer by putting in all the numbers

$$a + b = c + d$$
$$3 + 6 = 4 + 5$$
$$9 = 9$$

The same process is employed to discover the value of any term.
To sum up,

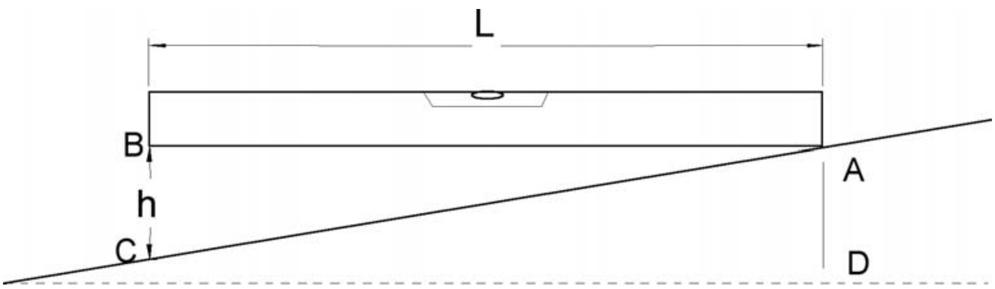
- isolate the required term ONE STEP AT A TIME
- always DO THE SAME TO BOTH SIDES,
- then replace the letters by any known numbers.

MEASURING GRADIENTS

It is quite common to require the incline, decline or camber angle of a road surface. Scale drawings should include such detail as a matter of course. It should always be included where it is relevant to the incident and/or is required in certain circumstances in order that calculations can be carried out, (*eg,* 'critical speed', skidding or rolling on an incline etc).

There are several devices available commercially designed to measure angles. One in common use is the 'Inogon' inclinometer. This has a vernier scale that allows the angle of a gradient to be read directly in degrees.

However, the following describes a simple method by which slope, including camber angles, can be measured using an engineer's level, (a 'spirit level'), and a steel measuring tape or ruler.



One end of the level is placed on the road surface and the level is held horizontal. The vertical distance from the other end of the level to the road surface is then measured. Measurements should be taken at several points along and/or across the road in order to minimise the possibility of erroneous information being used due to, say, road defects in one particular location. Care should be taken in the measuring of the height. Two persons may be required to ensure that the procedure is carried out accurately as it can prove difficult to hold the level sufficiently steady and measure the height at the same time.

The angle of the incline, decline or camber, is angle 'ACD'. As 'AB' and 'DC' are parallel, angle 'ACD' = angle 'BAC'.

The length of the level, '*L*', is easily measured and '*h*' is already known as that is to be measured. Therefore, if the angle 'BAC' is considered, the sides opposite and adjacent are known.

$$\text{Tan of angle} = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\text{Tan BAC} = \frac{h}{L}$$

$$\text{Angle BAC} = \text{Tan}^{-1}\left[\frac{h}{L}\right]$$

Example:

Length of level (*L*) = 120cms
Height (*h*) = 1.2cms

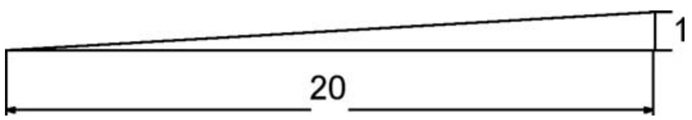
$$\text{Angle} = \text{Tan}^{-1}\left[\frac{1.2}{120}\right]$$
$$= \text{Tan}^{-1}\ 0.01$$
$$= 0.57°∞$$

It is worth noting at this point that when applying the above to '*critical speed*' calculations or to corrections to coefficients of friction on different inclines, the value required in the equation is the tangent of the angle. This means that $\frac{h}{L}$ can be put into the equation without further modification.

Note: Tan^{-1} means '*the angle whose Tan is....*'
eg, $\text{Tan}^{-1}\ 1 = 45°$∞
$\text{Tan}^{-1}\ 0.3 = 16.7°$∞

Other ways to express gradients

Older road signs show the gradient of a hill in terms of 1 in ?, but matters are in a transitional stage and newer signs show the gradient in terms of %age grade. It is important to be able to convert from one term to another. What the term 1 in ? actually means is that there is a rise of 1 unit for every 'x' units along the horizontal. In the example shown the horizontal measurement is 20 units. It should be apparent that the ratio would give us the tangent of that angle.



The following tables will show how to convert from one method of expressing a gradient to another.

Gradient Conversion Guide

To convert from 1 in 'x' to degrees

$$\frac{1}{x} = \text{Tan}\ \theta$$

e.g. To convert 1 in 20 to Degrees

$$\frac{1}{20} = \text{Tan}\ \theta$$

$$\text{Tan}\ \theta = 0.05$$

$$\text{Tan}^{-1}\ 0.05 = 2.9°$$

To convert from degrees to 1 in '*x*'

$$\left(\frac{1}{\text{Tan}\ \theta}\right) = 1 \text{ in '}x\text{'}$$

e.g. To convert 2.9° to 1 in '*x*'

$$\left(\frac{1}{\text{Tan } 2.9}\right) = 1 \text{ in '}x\text{'}$$

$$= 1 \text{ in } 20$$

To convert from degrees to percentage grade

$$\text{Tan}\ \theta \times 100 = \text{percentage grade}$$

e.g. To convert 2.9° to percentage grade

$$\text{Tan } 2.9° \times 100 = \%$$

$$= 5\%$$

To convert from percentage grade to degrees

$$\text{Tan}^{-1}\left(\frac{\text{percentage grade}}{100}\right) = \text{Degrees}$$

e.g. To convert 5% to degrees

$$\text{Tan}^{-1}\left(\frac{5}{100}\right) = \theta$$

$$= 2.9\%$$

To convert from percentage grade to 1 in '*x*'

$$\frac{100}{\text{percentage grade}} = 1 \text{ in '}x\text{'}$$

e.g. To convert 5% to 1 in '*x*'

$$\frac{100}{5} = 1 \text{ in '}x\text{'}$$

$$= 1 \text{ in } 20$$

To convert from 1 in '*x*' to percentage grade

$$\frac{100}{X} = \text{percentage grade}$$

e.g. To convert 1 in 20 to percentage grade

$$\frac{100}{20} = \%$$

$$= 5\%$$

Terminology

The following are NOT definitions. They are simply explanations of terminology the student may be confronted with.

Camber The lateral curvature - in a vertical plane - of the road surface when viewed in cross-section. The curvature is usually built-in as an aid to drainage of surface water.

Cross-fall The lateral inclination of the road surface as a result of the general slope of the surrounding land (*eg,* on a hillside). When the surrounding land has a general incline, it is cheaper to leave the lateral fall of the road to act as an aid to drainage rather than construct a cambered profile.

Gradient A slope in any longitudinal or lateral direction. Also the comparison of rise to run (expressed as 1:16, 6.25% or 3.5°).

Incline An uphill slope. May be expressed as per gradient.

Decline A downhill slope. May be expressed as per gradient.

Positive Relative to slopes, means either an uphill incline or a gradient which tends to cause a vehicle to slow down.

Negative Relative to slopes, means either a downhill slope or a gradient which tends to cause a vehicle to speed up.

Crown The highest portion of the cross-section of a cambered carriageway.

ACCEPTABLE PRECISION

Results of any calculations should not be quoted to more significant figures than the precision of the measurement justifies.

Thus, if the error in the result is ± 10% there is no justification for reporting the result to more than two significant figures. Suggested precision to be used in accident investigation cases are set out below.

Velocity

From skidmarks – the nearest whole number,
eg, 11.4ms^{-1} is reported as 11ms^{-1} and 35.7mph is reported as 36mph.

From impact damage – in multiples of 5ms^{-1} or 5mph,
eg, 16.4ms^{-1} is reported as 15ms^{-1} and 28.5mph is reported as 30mph.

Distance (calculated)

Below 10m – the nearest whole number,
eg, 5.8m is reported as 6m.
10m or above – to two significant figures,
eg, 77.6m is reported as 78m.

(NB These limits apply to calculated distances. It should be possible to measure actual distances of less than 30m to ± 0.1m. The major difficulty is deciding where to start and finish measuring.)

Time

Below 1s – one decimal place,
eg, 0.63s is reported as 0.6s.
Between 1s and 10s – the nearest half second,
eg, 4.43s is reported as 4.5s and 9.12s is reported as 9s.
Over 10 seconds – two significant figures,
eg, 15.7s is reported as 16s.

Calculations should be carried through to at least two decimal places and should then be rounded off at the end.